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LETTER TO THE EDITOR

Scaling of rough surfaces: effects of surface diffusion

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Abstract. Surface properties of a random deposition model in which the effects of surface diffusion are taken into account is studied in two dimensions. It is found that although the deposition bulk and the surface mass do not have fractal properties, the width of the surface is a self-affine fractal exhibiting non-trivial scaling with the surface height and the system size. The scaling results are found to agree with the scaling form proposed by Family and Vicsek for the ballistic deposition model, but with different exponents. This implies that random deposition with surface diffusion is in a different universality class from ballistic deposition and the simple random filling process.

Aggregation and growth processes [1] have been topics of considerable recent interest. One of the main characteristics of a growth process is the existence of an evolving interface separating the growing material from its environment. The study of this interface is of interest for two reasons. First, much of the complexity of non-equilibrium growth processes lies in the dynamics of the moving interface. Consequently a complete understanding of aggregation, dendritic solidification and related growth processes requires an understanding of their surface properties [1]. Second, characterisation of surface structure of growth processes is of practical interest due to the applications of surfaces in a wide variety of areas in science and technology. For these reasons, considerable attention has recently been focused on the problem of surface structure in aggregation [2, 3] and growth [4] models.

In this letter I report on the studies of the surface properties of a random deposition model which is a simplified representation for vapour deposition on a cold substrate [5]. In the random deposition model particles simply 'rain' down onto a substrate [3]. Particles move along straight line trajectories until they reach the top of the column in which they were dropped, at which point they stick to the deposit and become part of the aggregate. I will consider a modified random deposition model here, in order to account for the finite surface diffusion which exists in most of the realistic situations. In this model I allow a deposited particle to diffuse around on the surface within a prescribed region about the column in which it was dropped until it finds the column with the smallest height. At this point the particle sticks to the top of that column and becomes part of the aggregate. In the absence of this diffusive motion, the process is a random filling process in which there is no correlation between columns. Accordingly, the fluctuations in the column heights obey a Poisson process [3]. As we shall show below, with the introduction of surface diffusion the surface becomes smoother and,

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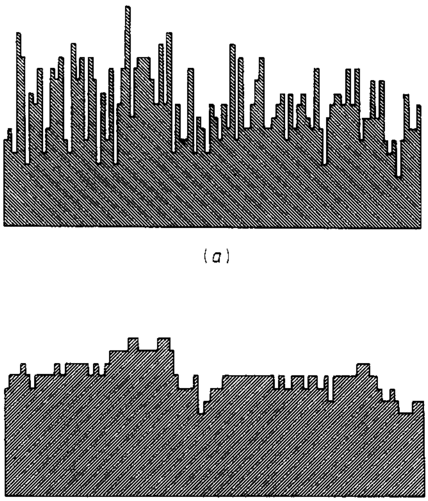
due to the correlation set-up by the surface diffusion, surface properties exhibit non-trivial scaling behaviour.

I have studied the random deposition model on a square lattice in which particles are deposited from above onto a line of L sites representing the initial nucleation seeds. I employed periodic boundary conditions so that columns i and $i+L$ are equivalent. I varied the distance over which a newly arriving particle was allowed to diffuse around. Since the results were found to be independent of this length, I only report the results for nearest-neighbour diffusion, i.e. a particle dropped in column i sticks to the top of column i, $i+1$ or $i-1$, depending on which of the three columns has the smallest height.

Examples of deposits with and without surface diffusion are shown in figure 1, where the smoothing effect of the surface diffusion can be readily seen even in this



(*a*)

(*b*)

Figure 1. Surface structure of deposits in (*a*) the random filling model and (*b*) in the random filling model with surface diffusion.

small scale simulation. Obviously, in random deposition there are no irregularities in the density of the bulk of the aggregate, because no empty spaces can form inside the deposit. Thus, the mass of the aggregate is compact and not a scale-invariant fractal [6]. The surface, however, is rough.

A measure of surface irregularities can be obtained from the width of the surface [3, 4]. I have determined the surface thickness from the variance [3]

$$\sigma = \left(\frac{\Sigma_i (z_i - \bar{z})^2}{N_s} \right)^{1/2} \tag{1}$$

where z_i is the height of the ith column, $\bar{z} = \Sigma_i z_i / N_s$ is the mean deposition height and N_s is the number of surface sites. In the deposition models studied here, $N_s = L$. The surface thickness $\sigma(L, h)$ depends on the system size L and the 'effective' deposition height h, defined as the number of deposited particles per seed site [3].

The simulation results in figure 2 show the dependence of $\sigma(L, h)$ on h for random deposition models with nearest-neighbour surface diffusion. The height of the column

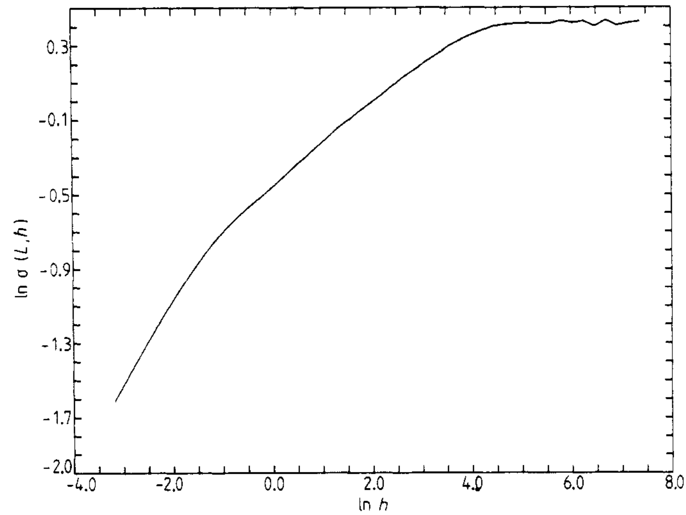


Figure 2. Dependence of the surface width $\sigma(L, h)$ on the effective deposition height h for $L = 48$. The results are averaged over 1000 independent simulations.

in the case with no diffusion follows a Poisson process and correspondingly the surface width σ diverges with the square root of h, independent of L [3]. In contrast, three separate regions are found in the case of deposition with diffusion as can be seen in figure 2. Initially, before a single layer of particles has been deposited, the diffusion process is unimportant, because it does not change the random placing of the particles on the seed particles and consequently σ varies as the square root of h. For intermediate values of h, i.e. beyond a monolayer but for $1 \ll h \ll L$, the surface diffusion tends to smooth out the surface and σ varies with h as

$$\sigma(L, h) \sim h^{\alpha} \qquad 1 \ll h \ll L. \tag{2}$$

The values of α for various L are tabulated in table 1. Asymptotically they appear to converge to a value of $\sigma = 0.25 \pm 0.01$. In the third region, $h \gg L$, the surface thickness saturates to a constant value $\sigma(L, \infty)$ depending on L. The dependence of σ on L is shown in a log-log plot of $\sigma(L, \infty)$ against L in figure 3. The straight line through the data points indicates that $\sigma(L, \infty)$ varies as

$$\sigma(L, \infty) \sim L^{\beta}. \tag{3}$$

From the slope of the straight line fits through the data I estimate $\beta = 0.48 \pm 0.02$. Since σ is independent of L in the random filling process, the non-trivial scaling of σ with h and L is due to the correlations generated by surface diffusion.

Table 1. Values of the exponent α defined in equation (3) for various L.

L	α
24	0.200
48	0.224
96	0.235
192	0.247
384	0.248

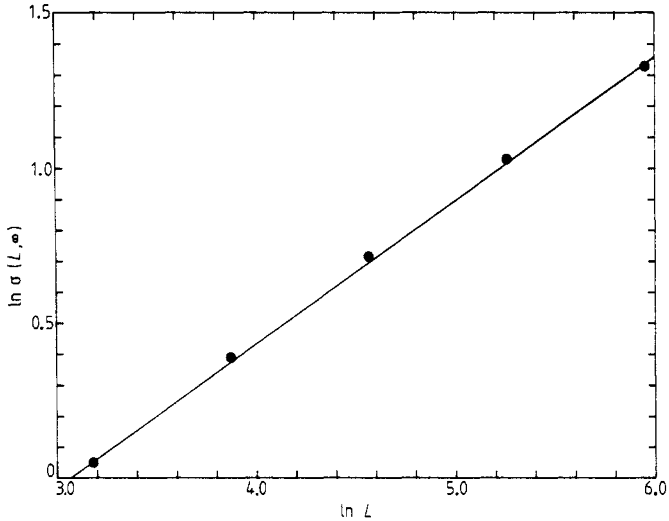


Figure 3. The logarithm of the saturation value of $\sigma(L, h)$ is plotted against the logarithm of L. The straight line through the data points indicates that $\sigma(L, \infty)$ scales as L^{β}.

The scaling results (2) and (3) for the dependence of σ on h and L is analogous to the results first obtained for the ballistic deposition model [3], but with different values of α and β. On the basis of results (2) and (3) for ballistic deposition, Family and Vicsek [3] proposed the following scaling form:

$$\sigma(L, h) = L^{\beta} f(h/L^{\gamma}) \tag{4a}$$

with $\gamma = \beta/\alpha$ and the scaling function $f(x)$ defined by

$$f(x) \sim \begin{cases} x^{\alpha} & x \ll 1 \\ \text{constant} & x \gg 1. \end{cases} \tag{4b}$$

In figure 4 I have plotted $\sigma(L, h)/L^{1/2}$ against h/L^2 for various L. The results collapse to a single scaling function in agreement with the scaling form (4). Similar data collapsing was obtained by Family and Vicsek for the ballistic aggregation model [3], but with $\alpha = 0.30 \pm 0.02$, $\beta = 0.42 \pm 0.03$ and $\gamma = 1.40 \pm 0.20$.

It is important to note that although surface diffusion changes the exponents in random deposition, I do not expect them to change in ballistic deposition. Introduction of surface diffusion in this model only amounts to a change in length scale and does not change the exponents. Therefore, the random deposition model with surface diffusion belongs to a different universality class from ballistic deposition and the random filling models.

An important implication of the above scaling results is that the two length scales in the problem, L and h, must be scaled by different ratios in order to keep the σ scale invariant. In order to motivate this unusual result I note that the random deposition model is a time-dependent process in which the effective height h plays the role of time t in the growth process [4]. Thus, the scaling form (4) for the surface width σ can be written in the form which is familiar from the dynamic scaling theory [7]

$$\sigma(L, t) = L^{\beta} f(t/L^{z}) \tag{5}$$

if we set $\gamma = z$.

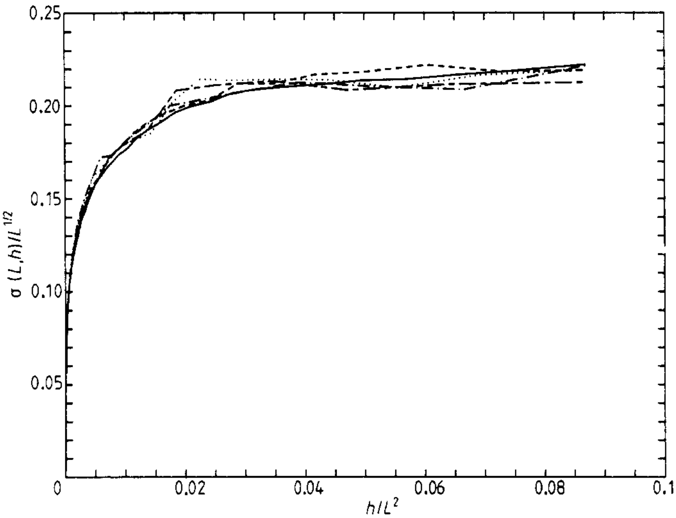


Figure 4. Scaling plot showing that the data for $\sigma(L,h)L^{-1/2}$ plotted against h/L^2 for various L fall on a single curve supporting the scaling form (5) with $\beta = \frac{1}{2}$ and $\gamma = 2$; (—-—): $L = 384$; (— · — · —): $L = 192$; (· · · · ·): $L = 96$ (-----): $L = 48$; (——) $L = 25$.

An alternative description of the surface scaling result can be given in terms of the theory of self-affine fractals [6]. Mandelbrot [6] has argued that fractal sets in which different lengths in the problem must be scaled differently are not self-similar but self-affine. In contrast to the usual self-similar fractals where all lengths in the problems scale with the fractal dimension or a single exponent, in self-affine fractals different lengths have different scale factors leading to several non-trivial scaling exponents and fractal dimensions. Therefore, surfaces in deposition models are self-affine instead of self-similar fractals.

The scaling of rough surfaces in granular deposits has been treated analytically by Edwards and Wilkinson [8] and more recently by Sander [9]. These authors write down the following Langevin type of equation for the time evolution of the surface profile $s(\boldsymbol{r}, t)$:

$$\partial s/\partial t = D\nabla^2 s + \xi(\boldsymbol{r}, t) \tag{6}$$

where s is a single valued function giving the height of the surface at a point specified by the $(d-1)$-dimensional vector $\boldsymbol{r}$ at time t. The first term on the right-hand side of (6) represents the surface relaxation due to the finite diffusion constant D, and $\xi(\boldsymbol{r}, t)$ is a zero mean, random noise term in the flux. In the above expression s is measured relative to its average value rather than the base of the bin. In the absence of the surface diffusion, the process described by the Langevin equation is simply one of random filling of bins which leads to an increase of σ with square root of t, as discussed before.

Since equation (6) is linear, it is quite easy to solve it by Fourier transformations [8, 9]. By calculating the variance σ directly one finds that

$$\alpha = (3-d)/4, \qquad \beta = (3-d)/2 \qquad \text{and} \qquad \gamma = 2 \tag{7}$$

in d dimensions. In $d = 2$, the result $\alpha = \frac{1}{4}$, $\beta = \frac{1}{2}$ and $\gamma = 2$ are in excellent agreement with the simulations. This implies that the Langevin equation (6) captures the essential physics of the random deposition process in the presence of a finite surface diffusion. Very recently, Kardar *et al* [10] have argued that a non-linear term must be added to (6) in order to account for the ballistic aggregation [3] and the Eden model [4] results.

In conclusion, studies of the surface properties of a non-trivial aggregation model which is a realistic representation of some vapour deposition experiments were reported. It was found that, although the bulk and the surface mass do not have fractal properties, the width of the surface is a self-affine fractal [6] exhibiting non-trivial scaling with the surface height and the system size. The scaling results were found to be in agreement with the scaling form (4), first proposed by Family and Vicsek [3] for the ballistic deposition model. Finally, the values of the exponents α, β and γ determined from the simulations were found to agree with the predictions of a Langevin equation approach [8, 9] for random deposition of granular aggregates.

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